Workshop program (titles and abstracts)

Alessandra BIANCHI

Title: *Random walk in a Lorentz gas random media*

Abstract: *In this talk we consider a one-dimensional process in random environment, also known in the physical literature as Levy-Lorentz gas. The environment is provided by a renewal point process that can be seen as a set of randomly arranged targets, while the process roughly describes the displacement of a particle moving on the line at constant velocity, and changing direction at the targets position with assigned probability. We investigate the annealed behavior of this process in the case of inter-distances between targets having infinite mean, and establish, under suitable scaling, a functional limit theorem for the process. In particular we show that, contrary to the finite mean case, the behavior of the motion is super-diffusive with explicit scaling limit related to the Kesten-Spitzer process. The key element of the proof is indeed a representation of the consecutive "hitting times on the set of targets" as a suitable random walk in random scenery (joint work with M. Lenci and F. Pène).*

Paul-Eric CHAUDRU DE RAYNAL

Title: *Kolmogorov PDE on Wasserstein space and McKean-Vlasov SDE*

Abstract: *In this talk we will focus on the smoothing effects of the Kolmogorov equation on Wasserstein space. Such an equation describes the dynamic of the semi-group generated by the solution of a McKean-Vlasov SDE (i.e. whose dynamics depend on the law). We will see how such effects allow to restore weak and strong well posedness for the McKean-Vlasov SDE as well as propagation of chaos phenomenon for the related interacting particle system in mean field.*
Joint work with N. Frikha.

François DELARUE

Title: *GLOBAL SOLUTIONS TO THE SUPERCOOLED STEFAN PROBLEM WITH BLOW-UPS*

Abstract: *We consider the supercooled Stefan problem, which captures the freezing of a supercooled liquid, in one space dimension. A probabilistic reformulation of the problem allows to define global solutions, even in the presence of blow-ups of the freezing rate. We provide a complete description of such solutions, by relating the temperature distribution in the liquid to the regularity of the ice growth process. The latter is shown to transition between (i) continuous differentiability, (ii) Hölder continuity, and (iii) discontinuity. In particular, in the second regime we rediscover the square root behavior of the growth process pointed out by Stefan in his seminal paper from 1889 for the ordinary Stefan problem. In our second main theorem, we establish the uniqueness of the global solutions, a first result of this kind in the context of growth processes with singular self-excitation when blow-ups are present.*
Joint work with S. Nadtochiy (Chicago) and M. Shkolnikov (Princeton).

Hong DUONG

Title: *Variational formulation for degenerate and non-local PDEs*

Abstract: *In 1998, Jordan-Kinderleher-Otto (JKO) proved a remarkable result that the Fokker-Planck equation can be seen as a gradient ow of the Boltzmann entropy with respect to the Wasserstein distance. This result has sparked off a large body of research in the field of partial differential equations and others in the last two decades. Many evolution equations have been proved to have a Wasserstein gradient ow structure such as the convection and nonlinear diffusion, the Cahn-Hilliard equation, the thin-film equation and finite Markov chains, just to name a few.*

In this talk I will present several extensions of the JKO approach to degenerate and nonlocal PDEs.

This talk is based on collaborative works [2,3,4].

[1]. R. Jordan, D. Kinderlehrer, and F. Otto. The variational formulation of the Fokker-Planck equation. SIAM Journal on Mathematical Analysis, 29(1):1-17, 1998.

[2]. M. H. Duong, M. A. Peletier, J. Zimmer. Conservative-dissipative approximation schemes for ageneralized Kramers equation. Mathematical Methods in the Applied Sciences, 37(16), 2517–2540, 2014.

[3]. M. H. Duong and Hoang Minh Tran. On the fundamental solution and a variational formulation of a degenerate diffusion of Kolmogorov type. Discrete and Continuous Dynamical Systems- Series A (DCDS-A), 38(7): 3407-3438, 2018.

[4]. M.H.Duong and Y.Lu. An operator splitting scheme for the fractional kinetic Fokker-Planck equation, to appear in Discrete and Continuous Dynamical Systems-Series A, 2019.

Jean-François JABIR

Title: *Propagation of chaos results for a class of McKean-Vlasov models*

Abstract: *This talk will be dedicated to presenting some recent results on the quantitative study of the propagation of chaos property related to a certain class of McKean-Vlasov dynamics with interaction kernels satisfying a certain moments control.*

Aline KURTZMANN

Title: *Asymptotic behaviour of some self-interacting diffusions*

Abstract: *In this talk, we will define and describe some self-interacting diffusions X living on $\mathbb{R}^d$. These are solutions to SDEs with a drift term depending on the actual position of the process and its normalized occupation measure $\mu_t = \frac{1}{t}\int_0^t \delta_{X_s} ds$, assuming a confinement potential satisfying some conditions. These hypotheses on the confinement potential are required since in general the process can be transient, and is thus very difficult to analyze.*

Using stochastic approximation methods, we will study the asymptotic behaviour of X_t and its empirical measure μ_t in different cases. Finally, we will present some very recent results obtained with Del Moral and Tugaut. This concerns the exit-time of a domain for a self-interacting diffusion, where the Brownian motion of replaced by σB_t, for a constant σ.

Michela OTTOBRE

Title: *A Vicsek-type model for self-propelled diffusions*

Abstract: *The study of interacting particle systems of self-propelled particles has attracted attention for a few decades and has posed interesting challenges to standard paradigms in statistical mechanics. As is well known, such models are at the root of many biological phenomena, such as bacterial migration, flocking of birds etc. In this talk we will consider a new continuum model of Viksek-type. Such a model is non-linear PDE which is i) not in gradient form and ii) it is non-uniformly elliptic. Moreover, as typical in this framework, the dynamics exhibits multiple equilibria (stationary states). This is a joint work with P.Butta (La Sapienza, Rome), F. Flandoli (Scuola Normale, Pisa) and B. Zegarlinski (Imperial College).*

Grigorios PAVLIOTIS

Title: *Long time behaviour of McKean-Vlasov equations: phase transitions and fluctuations*

Abstract: *We study the McKean-Vlasov equation on the flat torus which is obtained as the mean field limit of a system of interacting diffusion processes enclosed in a periodic box. Equations of this type are used in several applications in statistical mechanics, opinion dynamics, collective behaviour, and stellar dynamics. Under appropriate assumptions on the interaction potential, we show that the system exhibits multiple equilibria which arise from the uniform state through continuous bifurcations. Furthermore, we attempt to classify continuous and discontinuous transitions for this system. We then apply our results to the noisy Kuramoto model. For this model, we also study the combined mean field/homogenization(diffusive) limit and we study fluctuations around the McKean-Vlasov (law of large numbers) limit, both below and above the phase transition. This is joint work with José Carrillo, Matias Delgadino, Rishabh Gvalani and André Schlichting.*

Gonçalo dos REIS

Title: *The Support of McKean-Vlasov equations: Brownian Motion case*

Abstract: *We present recent results on two problems. The first is the construction of a Quantization for Gaussian Rough paths. We focus entirely on Brownian motion but other signals can be considered. We demonstrate key properties and prove a collection of equivalent rates of convergence. Secondly, these Codebooks are used to approximate the Law of McKean Vlasov Equations driven by the Gaussian Rough Path via systems of interacting ODEs. This allows us to represent the Support of the Law of a McKean Vlasov Equation in path space without needing to know the law of the solution explicitly.*

Lukasz SZPRUCH

Title: *Mean-Field Langevin Dynamics and Energy Landscape of Neural Networks*

Abstract: *We present a probabilistic analysis of the long-time behaviour of the nonlocal, diffusive equations with a gradient flow structure in 2-Wasserstein metric, namely, the Mean-Field Langevin Dynamics (MFLD). Our work is motivated by a desire to provide a theoretical underpinning for the convergence of stochastic gradient type algorithms widely used for non-convex learning tasks such as training of deep neural networks. The key insight is that the certain class of the finite dimensional non-convex problems becomes convex when lifted to infinite dimensional space of measures. We leverage this observation and show that the corresponding energy functional defined on the space of probability measures has a unique minimiser which can be characterised by a first order condition using the notion of linear functional derivative. Next, we show that the flow of marginal laws induced by the MFLD converges to the stationary distribution which is exactly the minimiser of the energy functional. We show that this convergence is exponential under conditions that are satisfied for highly regularised learning tasks. At the heart of our analysis is a pathwise perspective on Otto calculus used in gradient flow literature which is of independent interest. Our proof of convergence to stationary probability measure is novel and it relies on a generalisation of LaSalle's invariance principle. Importantly we do not assume that interaction potential of MFLD is of convolution type nor that has any particular symmetric structure. This is critical for applications. Finally, we show that the error between finite dimensional optimisation problem and its infinite dimensional limit is of order one over the number of parameters.*

Etienne TANRÉ

Title: *On a toy network of neurons interacting through their dendrites*

Abstract: *Consider a large number n of neurons, each being connected to approximately N other ones, chosen at random. When a neuron spikes, which occurs randomly at some rate depending on its electric potential, its potential is set to a minimum value v_{min}, and this initiates, after a small delay, two fronts on the (linear) dendrites of all the neurons to which it is connected. Fronts move at constant speed. When two fronts (on the dendrite of the same neuron) collide, they annihilate. When a front hits the soma of a neuron, its potential is increased by a small value w_n. Between jumps, the potentials of the neurons are assumed to drift according to some well-posed ODE. We prove the existence and uniqueness of a heuristically derived mean-field limit of the system when the size of the network increase.*
We give a link between the non-trivial intensity w_N and the size of the longest increasing subsequence of an i.i.d. collection of points in the plan (see [DZ]).
Common work with N. Fournier (U. Sorbonne Univ.) and R. Veltz (Inria).
[DZ] Deuschel, J-D. and Zeitouni, O. Limiting curves for i.i.d. records. Ann. Probab. 23 (1995).
[FTV] Fournier, N., Tanré, E. and Veltz, R. On a toy network of neurons interacting through their dendrites, https://arxiv.org/abs/1802.04118, to appear in AIHP (2019).

Milica TOMASEVIC

Title: *On a McKean-Vlasov SDE with singular time and space interaction related to the doubly parabolic Keller-Segel system*

Abstract: *The Keller-Segel (KS) model for chemotaxis is a two-dimensional system of parabolic or elliptic PDEs. Motivated by the study of the doubly parabolic model using probabilistic methods, we give rise to a non linear SDE of McKean-Vlasov type with a highly non standard and singular interaction. Indeed, the drift of the equation involves all the past of one dimensional time marginal distributions of the process in a singular way. In terms of approximations by particle systems, an interesting and, to the best of our knowledge, new and challenging difficulty arises: at each time each particle interacts with all the past of the other ones by means of a highly singular space-time kernel.*

In this talk, we will analyze the above mentioned McKean-Vlasov SDE and the associated particle system in order to exhibit new well-posedness results for the fully parabolic KS model in the case of $d = 1$ *and* $d = 2$*. This is a joint work with D.Talay (Inria) and J-F. Jabir (HSE, Moscow).*

References:

[1] J.-F. Jabir, D. Talay et M. Tomasevic (2018). Mean–field limit of a particle approximation of the one-dimensional parabolic-parabolic Keller-Segel model without smoothing. Elec. Communications Probab. 23, paper 84, 1-14, 2018.

[2] D. Talay et M. Tomasevic (2018). A new McKean-Vlasov stochastic interpretation of the parabolic-parabolic Keller-Segel model: The one-dimensional case. Under minor revision.

[3] M. Tomasevic (2019). A new McKean-Vlasov stochastic interpretation of the parabolic-parabolic Keller-Segel model: The two-dimensional case. Preprint