# Multivariate Possibility Distributions


Charles Lesniewska-Choquet[1], Abdourrahmane M. Atto[1], Gilles Mauris[1], Gregoire Mercier

[1] LISTIC, EA 3703, University Savoy Mont Blanc, Polytech Annecy-chambéry - France



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## Abstract


- Multivariate data analysis through the Possibility theory
- Definition of a generalised family of multivariate elliptical possibility distributions
- Definition of a divergence measure between possibility distributions


## Proposition

**Framework**

- The probability-possibility transformation framework proposed in one-dimension [1]
- The multivariate elliptical probability distributions [2]
- The Mahalanobis distance relating multivariate data to their monovariate closeness measures [3]

**Main Contributions**

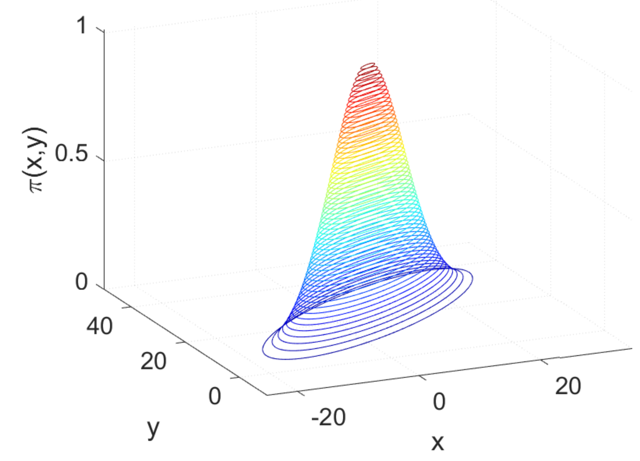
- Elliptical possibility distributions in n-dimension:

$$\pi(x) = 1 - \alpha_n \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})} \int_0^{a_x} r^{n-1} g_n(r^2) \mathrm{d}r,$$

where $a_x = \sqrt{(x-\mu)^T \Sigma^{-1} (x-\mu)}$ and $g_n$ is the density generator function.

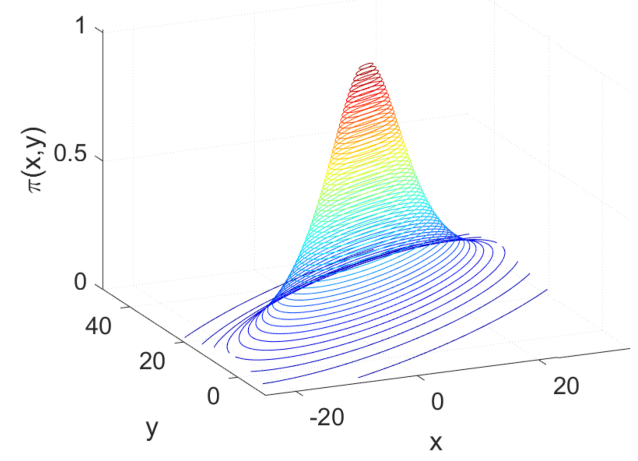
- Application to well-known probability distributions in 2-dimensions:

Normal distribution



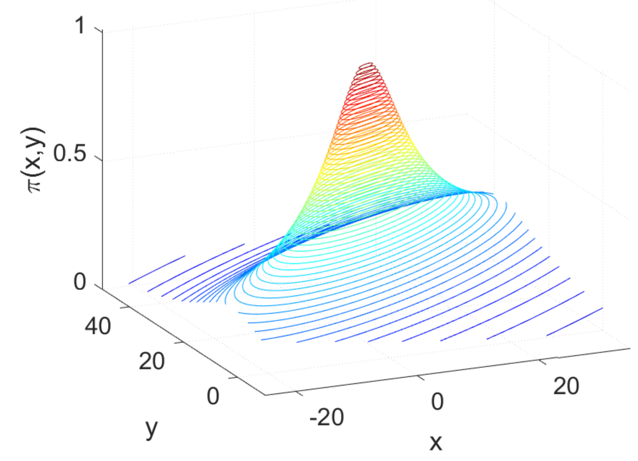

$$\pi_N(x) = e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$$

Student's t distribution




$$\pi_S(x) = \left(1 + \frac{1}{\nu}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)^{-\frac{\nu}{2}}$$

Cauchy distribution

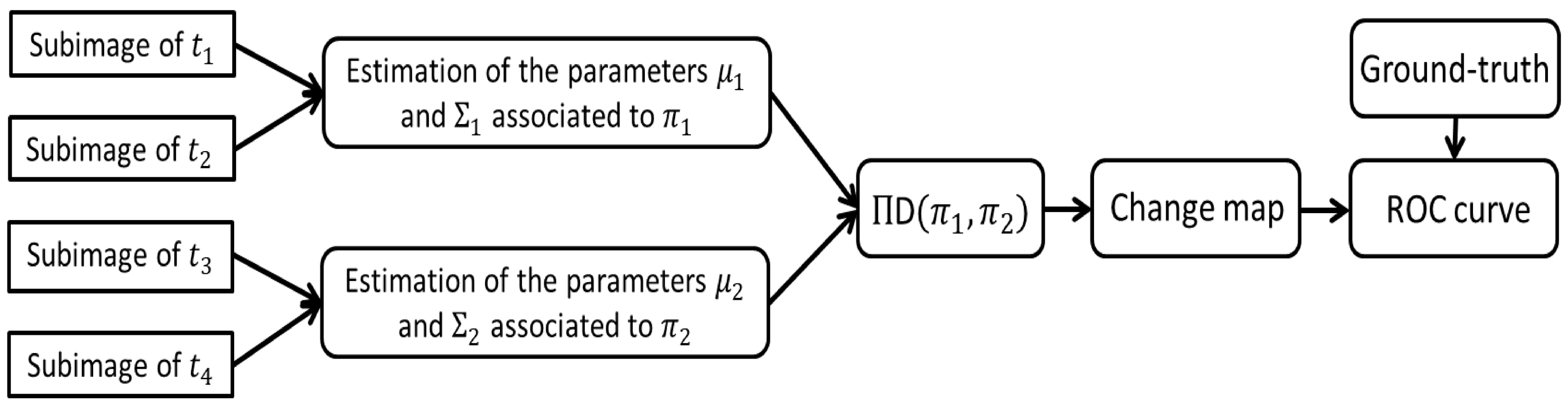


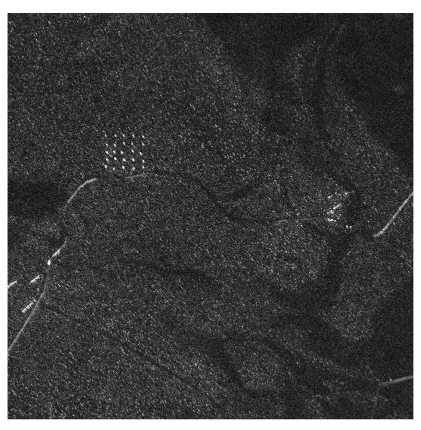

$$\pi_C(x) = \frac{1}{\sqrt{1+(x-\mu)^T \Sigma^{-1}(x-\mu)}}$$

- Analytical expression of the possibilistic divergence for the Normal distribution:

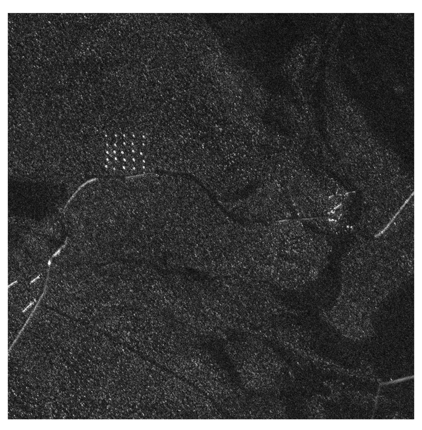
$$\Pi D(\pi_{N,1}, \pi_{N,2}) = \pi\sqrt{|\Sigma_1|}[\mathrm{tr}(\Sigma_2^{-1}\Sigma_1) + (\mu_1 - \mu_2)^T \Sigma_2^{-1}(\mu_1 - \mu_2) - 2] + \pi\sqrt{|\Sigma_2|}[\mathrm{tr}(\Sigma_1^{-1}\Sigma_2) + (\mu_2 - \mu_1)^T \Sigma_1^{-1}(\mu_2 - \mu_1) - 2]$$

## Application to real SAR Images: detection of vehicles concealed by foliage

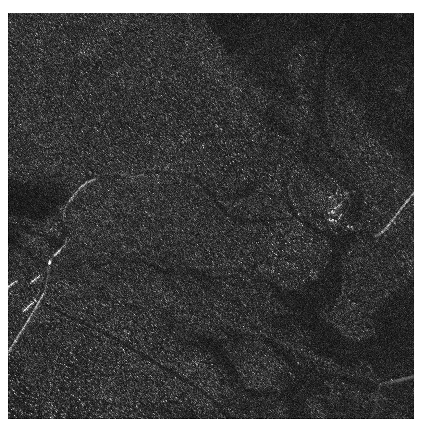


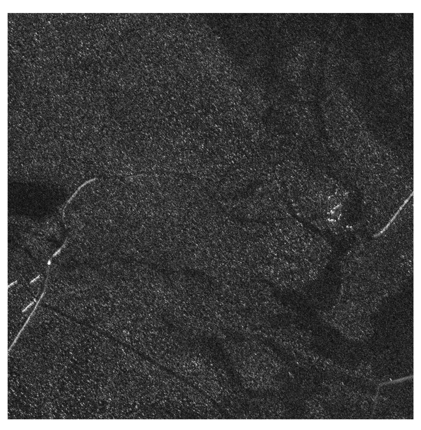
date *t1*



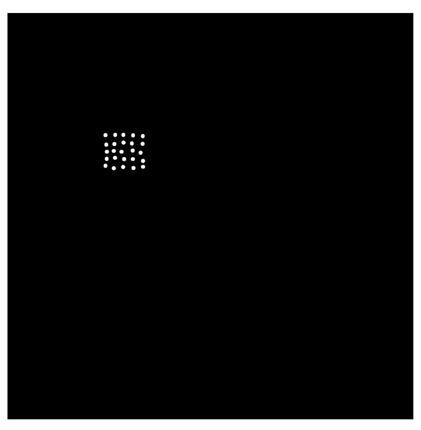
date *t2*



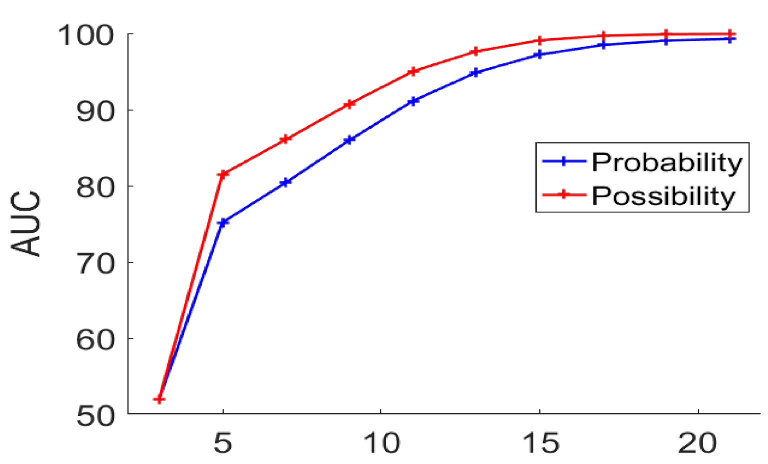
date *t3*



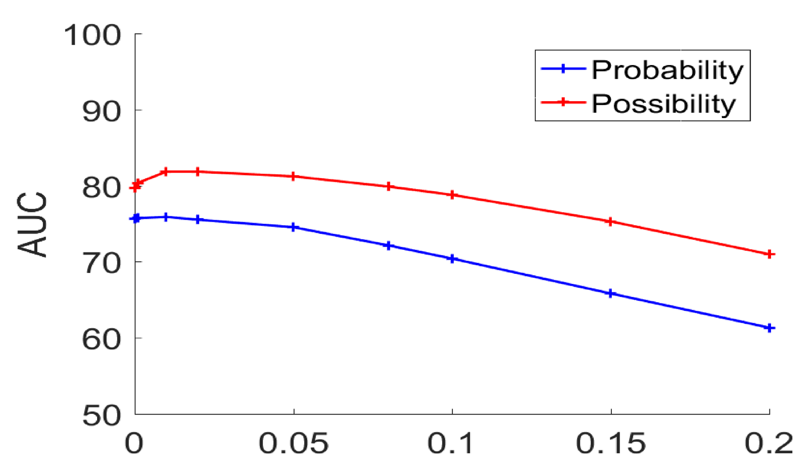
date *t4*



ground-truth




Size of the Sliding window




Variance of applied speckle noise

## Conclusions

We proposed a new family of elliptical possibility distributions thanks to the extension to the $n$-dimension of the continuous probability-possibility transformation in one dimension.

We emphasized the interest of a possibilistic framework in the field of multivariate data analysis especially when the data are noisy or their amount is insufficient to allow evaluating their characteristic parameters accurately.

## References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[2] S. Cambanis, S. Huang, and G. Simons, "On the theory of elliptically contoured distributions," *Journal of Multivariate Analysis*, vol. 11, no. 3, pp. 368–385, 1981.

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