Workshop program (titles and abstracts)

Yohann DE CASTRO

Title: *Exact tests on the mean of a Gaussian Process*

Abstract: *In this talk, we consider several ways of testing the mean of a Gaussian process with a special focus on their power against "sparse" alternatives. This investigation depicts the limit of the approaches based on evaluation of the process of very thin grids (even when joint with techniques inspired L1 recovery). A contrario, "gridless" methods accounting for the variation of the process (through the Hessian), are presented and seem to empirically outperform grid-based methods.*

References: Testing Gaussian Process with Applications to Super-Resolution (with J.-M. Azaïs and S. Mourareau), arXiv 1706.00679.

Marianne CLAUSEL

Title: *Gaussian random fields and anisotropy*

Abstract: *Textures in images can often be well modeled using self-similar random fields while they may at the same time display anisotropy. The present contribution thus aims at studying jointly selfsimilarity and anisotropy by focusing on two classic classes of anisotropic selfsimilar Gaussian fields.*

In a first part, we shall introduce a class of scale-invariant and anisotropic Gaussian fields and show how the anisotropy of the model can be characterized by the sample paths properties of the fields. We derive of these theoretical results a practical procedure to estimate anisotropy.

In the second part of the talk, we study a class of anisotropic and local self similar Gaussian random fields, and relate the orientation of the fields to the anisotropy properties of the texture. Notably, we use this preliminary study to define a new class of Gaussian fileds with prescribed orientation. Thereafter, we propose a practical procedure to perform the synthesis of these textures

Jean-Charles CROIX

Title: *Karhunen-Loève decomposition of Gaussian measures on Banach spaces*

Abstract: *The study of Gaussian measures on Banach spaces is of active interest both in pure and applied mathematics. In the Hilbert case, their decomposition is useful for simulation and data analysis (Principal Component Analysis, PCA). This relies on a spectral decomposition of the covariance operator, which provides a canonical decomposition of Gaussian measures on Hilbert spaces, the so-called Karhunen-Loève expansion. In this work, we intend to extend this result to Gaussian measures on Banach spaces in a very similar and constructive manner. In some sense, this can also be seen as a generalization of the spectral theorem for covariance operators associated to Gaussian measures on Banach spaces. In the special case of the standard Wiener measure, this decomposition matches with Paul LévyŠs construction of Brownian motion.*

Hassan MAATOUK

Title: *Constrained Gaussian processes: methodology, theory and applications*

Abstract: *Due to their flexibility, Gaussian processes (GPs) have been widely used in nonparametric function estimation. A prior information about the underlying function is often available. For instance, the physical system (computer model output) may be known to satisfy inequality constraints with respect to some or all inputs. We develop a finite-dimensional approximation of GPs capable of incorporating inequality constraints and observations (with and without noisy) for computer model emulators. It is based on a linear combination between Gaussian random coefficients and deterministic basis functions. By this methodology, the inequality constraints are respected in the entire domain. The mean and the maximum of the posterior distribution are well defined. Convergence of the method is proved and the connection with the spline estimates is done in the case of free-noisy data. This can be seen as an generalization of the Kimeldorf-Wahba (Kimeldorf and Wahba, 1971) correspondence between Bayesian estimation on stochastic process and smoothing by splines. A simulation study to show the efficiency and the performance of the proposed model in term of predictive accuracy and uncertainty quantification is included. Finally, a real application in insurance and finance to estimate a term-structure function and default probabilities is shown.*

Manfred OPPER

Title: *Gaussian processes for inference in stochastic differential equations*

Abstract: *Gaussian processes (GP) provide nonparametric approaches for inferring latent functions from data. In this talk I will discuss application of GPs to the case where the function models the drift (i.e. the deterministic driving force) in stochastic differential equations (SDE). I will discuss different scenarios: If the path of the SDE is observed densely in time, drift estimation reduces to simple GP regression. If observations are sparse in time, the GP likelihood can not be computed in an efficient way and we resort to an approximate EM algorithm where the unobserved paths of the SDE are treated as latent random variables. Since the complete data likelihood contains a continuum of infinite latent data we use a sparse Gaussian variational approximation to obtain a tractable MAP estimator. Finally, we discuss cases, where GPs can be used to estimate the drift using the empirical density of observations alone.*

Victor RABIET

Title: *Gaussian Random Field: simulation and quantification of the error, stationary case and application to some non-stationary cases*

Abstract: *We first show a construction and make an analysis of a simulation algorithm using fast Fourier transform for a stationary Gaussian Random field. This kind of algorithm can be found in (Wood and Chan, 1994) (and later in (Chan and Wood, 1999)) using a circulant embedding approach) and a decade later in (Lang and Potthoff, 2011), with a Fourier transform on a generalized white noise. The main point behind those algorithms is essentially the spectral representation, in the stationary case, of the covariance, when the latter is continuous.*

The first consequence of this common root is the error directly generated by the approximation used to compute the Fourier transform, here using a fast Fourier transform instead.

For the sake of applied matters, the quantification of such an approximation was indeed very important, especially if we want to use combinations of several simulated Gaussian Fields, for example, in order to simulate some non-stationary Gaussian Random fields.

It gives a very simple way to build non-stationary Gaussian Random fields as we will see in an application to a medical modelization of a layer of the human cornea; we will then discuss some ways to generalize this idea for the purpose of generating simulated non-stationary Gaussian Fields with this algorithm.