Multivariate Possibility Distributions

Charles Lesniewska-Choquet¹, Abdourrahmane M. Atto¹, Gilles MAURIS¹, Gregoire MERCIER

¹ LISTIC, EA 3703, University Savoie Mont Blanc, Polytech Annecy-chambéry - FRANCE

Acknowledgment: The work was supported by USMB and the PHOENIX ANR-15-CEE3-0012 grant of the French National Agency of Research.

Abstract

- Multivariate data analysis through the Possibility theory
- Definition of a generalised family of multivariate elliptical possibility distributions
- Definition of a divergence measure between possibility distributions

Proposition

Framework

- The probability-possibility transformation framework proposed in one-dimension [1]
- The multivariate elliptical probability distributions [2]
- The Mahalanobis distance relating multivariate data to their monovariate closeness measures [3]

Main Contributions

- Elliptical possibility distributions in n-dimension:
  \[
  \pi(x) = 1 - a_x = \int_0^{a_x} r^{n-1} g_n(r^2)dr
  \]
  where \(a_x = \sqrt{(x-\mu)^T \Sigma^{-1} (x-\mu)}\) and \(g_n\) is the density generator function.
- Application to well-known probability distributions in 2-dimensions:
  - Normal distribution
    \[
    \pi_N(x) = e^{\frac{1}{2}\left[(x-\mu)^T \Sigma^{-1} (x-\mu)\right]}
    \]
  - Student’s t distribution
    \[
    \pi_T(x) = \left(1 + \frac{1}{2} (x-\mu)^T \Sigma^{-1} (x-\mu)\right)^{-\frac{1}{2}}
    \]
  - Cauchy distribution
    \[
    \pi_C(x) = \frac{1}{\sqrt{1+(x-\mu)^T \Sigma^{-1} (x-\mu)}}
    \]
- Analytical expression of the possibilistic divergence for the Normal distribution:
  \[
  IID(\pi_N, \pi_N) = \pi \sqrt{\Sigma_1 \text{tr}(\Sigma_2^{-1} \Sigma_1) + (\mu_1 - \mu_2)^T \Sigma_2^{-1} (\mu_1 - \mu_2)^2 + \pi \text{tr}(\Sigma_2^{-1} \Sigma_1) (\mu_1 - \mu_2)^T \Sigma_1^{-1} (\mu_2 - \mu_1)^2}
  \]

Application to real SAR Images: detection of vehicles concealed by foliage

Conclusions

We proposed a new family of elliptical possibility distributions thanks to the extension to the n-dimension of the continuous probabilistic-possibility transformation in one dimension.

We emphasized the interest of a possibilistic framework in the field of multivariate data analysis especially when the data are noisy or their amount is insufficient to allow evaluating their characteristic parameters accurately.

References