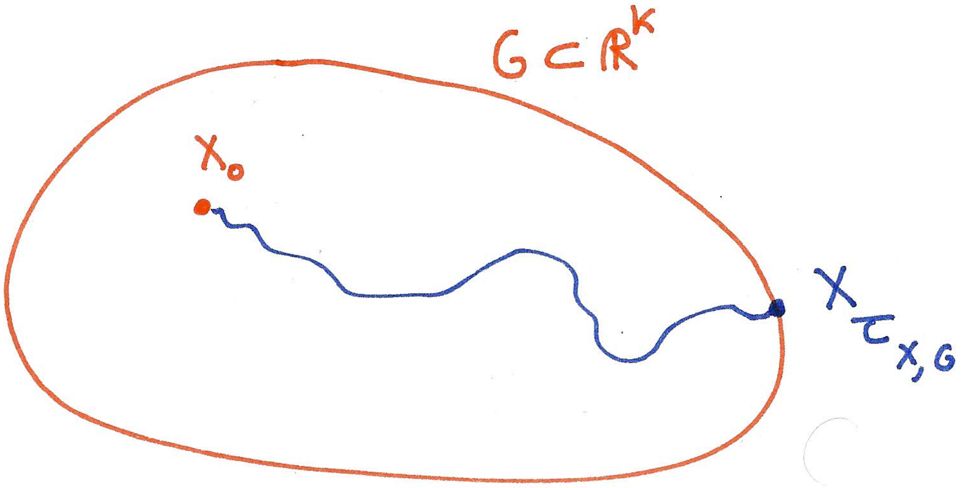
Exit problem of self-stabilizing diffusions

I) Introduction

(a) Exit problem

$(X_t)_{t\geq 0}$ stochastic process

* Diffusion
* Jump process
* PDMP
* ...



$$\tau_{x,G} := \inf\{t \geq 0 : X_t \notin G\}$$

Goal:

* Exit time : $\tau_{x,G}$?
** Exit location : $X_{\tau_{x,G}}$? ($\in \partial G$?)

(b) Freidlin-Wentzell theory

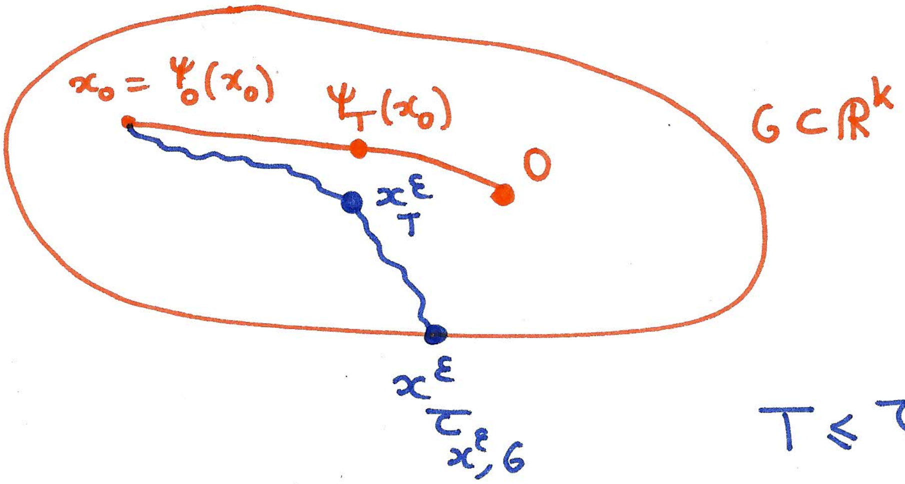
$$x_t^\varepsilon = x_0 + \sqrt{\varepsilon}\,\beta_t - \int_0^t \nabla U(x_s^\varepsilon)\,ds$$

$$\psi_t(x_0) = x_0 \qquad - \int_0^t \nabla U(\psi_s(x_0))\,ds$$

$$\begin{pmatrix} x_0 \in \mathbb{R}^k \\ \nabla^2 U \geqslant \lambda > 0 . \quad U \geqslant U(0) = 0 \end{pmatrix}$$

Lemma

$$\forall T > 0, \quad \lim_{\varepsilon \to 0} \mathbb{E}\Big\{ \sup_{[0;T]} \| x_t^\varepsilon - \psi_t(x_0) \| \Big\} = 0 .$$



$$T \leqslant \tau_{x^\varepsilon, G}$$

- Immediately, if $\{\psi_t(x_0) ; t \geqslant 0\} \subset G$, we deduce

$$\lim_{\varepsilon \to 0} \mathbb{P}\{\tau_{x^\varepsilon, G} \leqslant T\} = 0 .$$

- Reciprocally, if $\exists T_0 > 0$ s.t. $\psi_{T_0}(x_0) \notin \overline{G}$, we have

$$\lim_{\varepsilon \to 0} \mathbb{P}\{\tau_{x^\varepsilon, G} \leqslant T_0\} = 1 .$$