Thesis proposal: Limit properties of Langevin models and applications to cooperative multi-agent dynamics

The aim of this thesis is to investigate the asymptotic behaviour, at different characteristic scales (long-time, zero noise and large population), and the metastability properties of a class of stochastic kinetic models depicting cooperative multi-agent systems. Broadly, these models consist in an ensemble of N-coupled Langevin diffusion processes given by:

$$\begin{aligned} X_t^{i,N} &= X_0^{i,N} + \int_0^t U_s^{i,N}\,ds, \\ U_t^{i,N} &= U_0^{i,N} - \int_0^t W(X_s^{i,N}, U_s^{i,N})\,ds - \sum_{j=1}^N \int_0^t \xi_s^{i,j,N}\left(U_s^{i,N} - U_s^{j,N}\right)ds + \int_0^t \sigma_s^{i,j,N}\,dB_s^i, \qquad t \geq 0,\ i = 1, \cdots, N, \end{aligned} \tag{1}$$

each pair $(X_t^{i,N}, U_t^{i,N})$ modelling the position and velocity of a main component - or agent - of the system at a given time t. The driving noises $(B^1, \cdots, B^N)$ are set as a family of independent multi-dimensional Brownian motions while the derive function W represents a common exogenous force acting on the motion of each individual of the system. Interactions, producing local or global correlations between agents, are modelled by the communication processes $\xi^{i,j,N}$ and $\sigma^{i,j,N}$.

Among all the possible situations which can be featured from (1), a particular interest lies in the case where interactions create a self-stabilising effect leading to a *agglomeration-and-alignment* phenomena of the system at large-time. More specifically, our particular interest starts with the (deterministic) Cucker-Smale model describing the evolution of populations where a collective behaviour, in the form of a swarm, emerges at large time. Mathematically, the model formulates as a special case of (1) where interactions are reduced to the sole communication:

$$\xi_t^{i,j,N} = K\left(\rho^2 + ||X_t^{i,N} - X_t^{j,N}||^2\right)^{-\gamma}\left(U_t^{i,N} - U_t^{j,N}\right), \qquad t \geq 0.$$

Under some specific assumptions on the model parameters, K, ρ, γ, and on the initial positions and velocities of the population, the Cucker-Smale model successfully captures the emergence at large time $t \to \infty$, of a global alignment of the population ($\max_{i,j} ||U_t^{i,N} - U_t^{j,N}|| \to 0$) and an agglomeration of its individuals ($\sup_{i,j} ||X_t^{i,N} - X_t^{j,N}|| < \infty$). Since its introduction, the model has been extended to integrate more complex situations issued from social sciences and econophysics - for instance, to account for the presence or emergence of distinct leaders or supplementary dispersion effects due to the media. For the latter, the presence of a stochastic perturbation in the model significantly impacts swarming, whether the perturbation appears through a basic additive private noise, e.g.

$$X_t^{i,N} = X_0^{i,N} + \int_0^t U_s^{i,N}\,ds, \quad U_t^{i,N} = U_0^{i,N} - \frac{1}{N}\sum_{j=1}^N \int_0^t \frac{K}{\left(\rho^2 + ||X_s^{i,N} - X_s^{j,N}||^2\right)^{\gamma}}\left(U_s^{i,N} - U_s^{j,N}\right)ds + \sqrt{2}\sigma B_t^i,$$

or in a multiplicative form or as a common noise. Depending on the way noise is casted, different form of probabilistic interpretation of the swarm can be devised.

While agglomerating systems have been well-studied in over-damped or in zero-noise regimes, little is effectively known for the actual dynamics (1) notably an agglomerating-alignment setting. The principal focus of the thesis will be so to study this limit behaviour with a metastability viewpoint, and estimating how the first time that alignment and a (fixed) agglomeration are destroyed or restored in the small-noise limit, in the situations of a finite or an asymptotically large ($N \to \infty$) population. In this context, stochastically perturbed Cucker-Smale models with uniformly convex and multistable exogenous potential (modelling distinct attracting sources or asymptotic leading directions) will be first considered. For this specific situation, a first step will be to extend, from overdamped situations to a more general kinetic framework, Freidlin-Wentzell's approach to Arnhennius-Eyring-Kramers' exit time laws and recent applications of this approach for establishing qualitative small-noise asymptotic of the collision-times of particle systems. As very few papers has been dedicated tokinetic cases, this step will itself require to extend a variety of existing results on the metastability properties of perturbed dynamical systems to a completely new framework. The existence of an appropriate Kramers-Smolushovski scaling will be also addressed, with the view of establishing a consistent stability theory connecting the overdamped models to their kinetiv counterparts.

Practical informations.

Advisors: The thesis will be jointly supervised by Dr. Jean-François Jabir and Dr. Julian Tugaut. The latter and his institute will be the main supervisor and host for the PhD candidate. Co-supervision will be conducted on site with the frequent visit of the second advisor in Saint-Étienne and via long-distance meetings. Opportunity to research stays abroad will be offered to the candidate.

Julian Tugaut is *Maître de Conférences* at the Université Jean Monnet at Saint-Étienne since 2013 (*Hors Classe* since 2022). He is currently the principal researcher of the ongoing ANR, METANOLIN, project, a 2021-2023 Sakura program with Japan and a Asgard-research program with Norway. He is the author of more than 25 articles in renowned scientific journals and has collaborated with researchers in France, Europe and abroad. Jean-François Jabir is currently *Assistant Professor* at the Higher School of Economic, Moscow. He previously managed and collaborated in research and educational programs in Chile (CONICYT, ANID) and in France (INRIA, ADEME).

Place: The PhD thesis will be hosted at the Université Jean Monnet (UJM). The UJM is a multi-disciplinary university and the Saint-Étienne campus of Camille Jordan Institute covers a wide range of mathematics (probability, statistics, PDE, numerical analysis, number theory and algebra). Several ongoing projects provide excellent condition and support for junior researchers. Moreover, administrative support for relocation in France, and more precisely in Saint-Étienne, will be provided if necessary. While teaching appointments are not mandatory albeit, tutoring and seminar assistance position (in french) will be offered to the PhD candidate.

Requirements: This thesis is addressed to students with a strong background on probability theory, stochastic modelling and stochastic analysis. Skills in dynamical systems, limit theorems for stochastic processes and large deviations techniques will be appreciated.

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Procedure of application: Candidates are required to send to the contact addresses: a CV where Bachelor and Master grades, specific skills in probability theory, stochastic analysis, and ideally large deviations principles, are detailed; a motivation letter (of no more than two pages) mentioning career projects, scientific interests and adequacy to the subject and, potentially, an integration project in UJM. Recommendation letters are welcome.

Deadline to apply: Monday 17th of April 2023, at 17h00, Paris hour.

Interview: Interview will take place in Saint-Étienne on the 27th of April in an off-line or on-line format; students selected for the interview will be informed of the off-line location two weeks prior to the meeting. For nationals, traveling and living expenses for the interviews will be covered (up to a limited amount).